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Interaction Analysis and Geometric Interconnection Decoupling for Networks of Process Systems

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The effects of interactions between nonlinear subprocesses on the stabilizability of plantwide systems via the concept of dissipative systems are studied. Conditions for which controlled variables of each interconnected subprocess can be driven to and maintained at their desired values are established through the application of interconnection decoupling techniques. The resulting decoupling feedback law encodes the interaction effects between subprocesses and determines the required information structure for achieving desired control performance using distributed control laws. The proposed constructive approach leads to new criteria for the selection of manipulated and controlled variables that guarantee plantwide stability. © 2013 American Institute of Chemical Engineers *AIChE J*, 59: 2795–2809, 2013
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Introduction

In chemical process control, plantwide systems refer to networks of processes interconnected by mass and energy streams.[1] The design of centralized multivariable control algorithms for large-scale plantwide problems, which may involve up to thousands of control loops,[2] is generally not feasible in practice. Instead, a hierarchical control structure is usually implemented to accomplish effective plantwide control.[3] The idea is to delegate different tasks to control loops (which may be of different types) at each hierarchical level. At the top of the hierarchy, decisions regarding optimal operating conditions are made, whereas at the base, local decentralized controllers are assigned to maintain the stability of the plantwide system and the quality of the end products. The effects of process interactions have to be considered at the latter, as they may interfere with the dynamics of each process and contribute to network destabilization. For example, adverse effects of plant interactions were reported with the integration of recycle streams.[4] In this article, we focus on the control systems at this base level and aim to study the effects of subsystem interactions on the overall stability of plantwide systems.

The importance and possible dire consequences of process interactions have prompted several attempts in the past to study and manage their effects by finding a proper network decomposition. The idea is to define a collection of subnetworks of a plantwide system such that there are limited or manageable interactions occurring among them. Analysis and control design can then be performed on each different, simpler subnetworks. In the articles by Baldea et al.,[5,6] for example, networks are decomposed into subnetworks according to different time scales. The control is then designed for each time scale. Zhu et al.,[7,8] considered the decomposition of networks according to the linear/nonlinear behaviors of their subsystems. Highly nonlinear processes are isolated, and nonlinear control strategies are implemented for these processes only, whereas relatively simpler linear controllers are designed for the linearly behaved processes. In the work by Antelo et al.,[9] the networks are decomposed according to convective and diffusive currents as well as mass and energy flows. In the context of this work, we shift from the problem of finding the appropriate decomposition and treat plantwide systems as natural networks of process systems as viewed by Ydstie.[1] Similar to the arguments made by Ikeda and Siljak,[10] however, the approach is not limited to this type of network decomposition. Possible decomposition and aggregation of subsystems can be performed to place the results proposed in this article for different types of network decompositions.

In addition to the difficulties posed by the interactions between subsystems, it is well known that chemical process systems are nonlinear, and their behaviors are governed by the fundamental laws of thermodynamics.[11] Building on these laws, the nonlinear models of many chemical process systems in a network consist of multiple states, most of

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which are interconnected to neighboring process systems, and are often underactuated. From a geometric control point of view, these observations imply that (1) each process system has "internal dynamics" that cannot be controlled due to the lack of degree of freedom and (2) the states corresponding to these internal dynamics, that is, the "internal states," have an effect on neighboring subsystems. The minimum phase property is useful[12] to assess the stability of internal dynamics in a single system. However, the effects of subsystems' internal dynamics over a network are less explored in the literature. The main focus of the present contribution is to study the stability of interconnected subsystems with internal dynamics in the context of plantwide systems, assuming throughout that the assigned controlled variables are successfully maintained at their desired equilibrium values. It is important to note that even when the controlled variables remain at their desired operating values, plantwide stability in the sense of Lyapunov may not be achieved. Unstable internal dynamics may lead to physically infeasible control law (e.g., if the control law depends on the internal dynamics and the latter are unstable)[13] and overall plantwide instability. In this article, the effects of internal dynamics on plantwide stability are investigated via dissipative systems theory.

The concept of dissipative systems in control was first introduced in the seminal article by Willems[14] and have found extensive applications in the area of robust and optimal control.[15] It was found that some of the most important dynamical properties of process systems, such as their stability and norm gain bound, can be inferred from their dissipativity.[16] Defined as an input–output property, dissipativity can be conveniently used in a network setting,[17] and, thus, is suitable for the study of large-scale systems (which can always be viewed as network systems), including plantwide systems. Application of dissipative systems theory to plantwide process control were explored, for example, in the articles by Rojas and Bao,[18] Xu and Bao,[19] and Setiawan et al.[20] Motivated by the constructive results on decentralized control by Han and Chen,[21] Guo et al.,[22] and Xie and Xie,[23] we seek to integrate geometric approaches into the existing dissipativity-based analysis of large-scale networks. In particular, the results by Ebenbauer and Allgöwer[24] are extended to formally develop dissipativity-based conditions that guarantees minimum phase property for large-scale systems.

The results obtained in this study on large-scale minimum phase property may be useful in the preliminary selection of manipulated/controlled variable pairings in plantwide control structure design.[3] They can be used to guarantee the internal stability of plantwide systems. They can also assist in geometric decentralized control design by isolating the effects of interactions between internal dynamics of different subsystems, thereby providing more flexibility in constructing a control law required to drive the controlled variables to the desired reference signals.

The proposed analysis is accompanied with a control design approach that ensures the convergence of the controlled variables. In a plantwide setting, this task is nontrivial as the interactions between subsystems may perturb the controlled variables. The possibility of stabilizing the controlled variables is studied in this article using decoupling techniques. In particular, a set of geometric conditions for the solvability of interconnection decoupling are established in this article by extending the disturbance decoupling methodologies by Daoutidis et al.[25,26] to the concept of interconnection decoupling. The obtained solution describes the structural requirement for each subsystem such that the interaction effects are decoupled from the assigned controlled variables under nonlinear state feedback control laws. The proposed analysis can, therefore, be used to make geometric assessments of the effects of system interactions. In the context of distributed control, it can be interpreted as a requirement on information exchange between subsystems[1] to achieve interconnection decoupling and, hence, stabilization of large-scale systems. The results are stated in graph theoretic perspective[17] and are reminiscent of the results by Daoutidis and Kravaris,[27] wherein the graph is used to characterize the geometric structure of nonlinear systems. Although the formal statement of the results is analytic, the solution is constructive and can be used for control design.

The remainder of this article is structured as follows. The class of large-scale systems studied in this article, and the preliminaries on the theory of dissipative systems are given in the "Preliminaries" section. Analysis of plantwide systems and the determination of large-scale minimum phase property are performed in the "Large-Scale Minimum Phase Analysis" section. In the "Interconnection Decoupling" section, the main results on the solvability of the interconnection decoupling problem are presented, using the theory of directed graphs. The main results of the article are illustrated in the "Illustrative Example" section with a network of polynomial systems resembling a biological network and a benchmark example of a process control network consisting of a reactor and a separator. Some important remarks and implications about our results are discussed in "Discussion" section. Finally, concluding remarks and future areas of study are given in the "Concluding Remarks" section.

Preliminaries

Class of systems

Throughout this article, a class of large-scale nonlinear systems Σ, composed of single input single output (SISO).* subsystems Σ_i ($1 \le i \le N$) is studied. Each Σ_i is given as

$$\Sigma_i : \begin{cases} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i + W_i(x_i)\tilde{u}_i \\ y_i &= h_i(x_i) \\ \tilde{y}_i &= \tilde{h}_i(x_i) \end{cases} \tag{1}$$

where the state $x_i = \mathrm{col}\left(x_{i,1}, \ldots, x_{i,n_i}\right) \in \mathbb{R}^{n_i}$, the manipulated variable $u_i \in \mathbb{R}$, and the controlled variable $y_i \in \mathbb{R}$, while $\tilde{u}_i \in \mathbb{R}^{m_i}$ and $\tilde{y}_i \in \mathbb{R}^{p_i}$ represent the interconnecting input and output, respectively. It is assumed that the vector fields $f_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$, $g_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$, $W_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i \times m_i}$, and the functions h_i and $\tilde{h}_i$ are sufficiently smooth, with $f_i(0) = 0$, $h_i(0) = 0$, and $\tilde{h}_i(0) = 0$. Define the interconnection topology matrix H as

$$\tilde{u} = H\tilde{y} \tag{2}$$

where $\tilde{u} = \mathrm{col}\left(\tilde{u}_1, \ldots, \tilde{u}_N\right)$ and $\tilde{y} = \mathrm{col}\left(\tilde{y}_1, \ldots, \tilde{y}_N\right)$. Hence, by partitioning the topology matrix, we have

The development presented in this article focuses only on SISO systems. This is mainly for the simplicity of geometric statements made in the sequel. One can generalize the results to multiple input multiple output (MIMO) systems using geometric nonlinear control approaches available, for example, in the monograph by Isidori.[28]

$$\tilde{u}_i = H_{ij}\tilde{y}_j \tag{3}$$

Physically interpreted in plantwide studies, $\tilde{u}_i$ consists of the inlet mass and energy convective flows into Σ_i from different units and $\tilde{y}_i$ is the total outlet mass and energy convective flows from Σ_i. The above topology matrix encodes the plantwide network structure in a way similar to Kirchoff convection matrix discussed by Hangos et al.[29]

In the sequel, we assume that for each Σ_i, there exist a global change of coordinates $(z_i, \zeta_i) = \phi_i(x_i)$, $\phi_i(0) = 0$, with the first ρ_i terms selected as follows

$$\begin{aligned} z_{i,1} &= h_i(x_i) \\ &\vdots \\ z_{i,\rho_i} &= \mathcal{L}_{f_i}^{\rho_i-1} h_i(x_i) \end{aligned} \tag{4}$$

where ρ_i denoted the strong relative degree† of Σ_i, and a static state feedback

$$v_i = \mathcal{L}_{g_i}\mathcal{L}_{f_i}^{\rho_i-1} h_i(x_i) u_i + \mathcal{L}_{f_i}^{\rho_i} h_i(x_i) \tag{5}$$

that globally transforms (1) into

$$\tilde{\Sigma}_i : \begin{cases} \dot{z}_{i,1} & = z_{i,2} + \Phi_{i,1}(z_i)\tilde{u}_i \\ & \vdots \\ \dot{z}_{i,\rho_i-1} & = z_{i,\rho_i} + \Phi_{i,\rho_i-1}(z_i)\tilde{u}_i \\ \dot{z}_{i,\rho_i} & = v_i + \Phi_{i,\rho_i}(z_i)\tilde{u}_i \\ \dot{\zeta}_i & = \psi_i(z_i, \zeta_i) + \Psi_i(z_i, \zeta_i)\tilde{u}_i \\ y_i & = z_{i,1} \end{cases} \tag{6}$$

Observe that (6) is a slight modification to the Byrnes–Isidori normal form (p. 143),[28] and the existence of transformation to such form is standard in nonlinear disturbance decoupling.[30] The precise geometric conditions guaranteeing the existence of the above coordinate transformation are given in the article by Byrnes and Isidori.[31] The last $n_i - \rho_i$ terms of the above representation, that is, $\zeta_i \in \mathbb{R}^{n_i-\rho_i}$, constitute the internal states of the system, and their dynamics are often referred to as the internal dynamics. In the above form, the stability of the internal dynamics is independent of the feedback control law v_i. For an isolated Σ_i (where $\Phi_i = 0$ and $\Psi_i = 0$), $\dot{\zeta}_i = \psi_i(0, \zeta_i)$ becomes the zero dynamics of that subsystem. If the zero dynamics is asymptotically stable, the system is said to be minimum phase.

In the case where $z_i = 0$ and $\Psi_i \neq 0$, the stability of the zero dynamics of each subsystem are dependent on their interactions. This effect is studied in this article using the concept of dissipative systems, reviewed next.

Dissipative systems theory

We first recall the definition of dissipative systems in control theory.

Definition 1. (Willems[14]). *A system with state* x*, input* u*, and output* y *is said to be dissipative with respect to a square integrable function* s*, called the supply rate, if there exists a positive semidefinite function $V : \mathbb{R}^n \to \mathbb{R}^+$, called the storage function, such that the following dissipation inequality is satisfied*

$$V(x(t_0), t_0) - V(x(t_1), t_1) \leq \int_{t_0}^{t_1} s(x(t), u(t), y(t))dt \tag{7}$$

In this article, we assume that we have, for each Σ_i, an autonomous, positive definite storage function, that is, $V(x) > 0$ for all $x \neq 0$ and $V(0) = 0$. Also note that unlike in the original formulation as stated by Willems[14] and Hill and Moylan[16] where the supply rate is defined only on the input–output space, it is allowed to be a function of states. This supply rate is a generalization of input-to-state stability (ISS) supply rate as presented by Sontag and Wang,[32] and the form is considered by Ebenbauer and Allgöwer[24] to study the minimum phase property of a system.

The sufficient and necessary dissipativity condition for minimum phase systems is given by Ebenbauer and Allgöwer[24] as follows.

Theorem 1. *(Ebenbauer and Allgöwer[24]). Consider a nonlinear system of the form*

$$\begin{aligned} \dot{x} &= f(x) + G(x)u \\ y &= h(x) \end{aligned} \tag{8}$$

$f : \mathbb{R}^n \to \mathbb{R}^n$, $G : \mathbb{R}^n \to \mathbb{R}^{n\times p}$, and $h : \mathbb{R}^n \to \mathbb{R}^p$ and $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ is the manipulated variable, and $y \in \mathbb{R}^p$ is the controlled variable. Assume that there exists a smooth global change of coordinate that transforms the system into the Byrnes–Isidori normal form.[28] Then, the system has the minimum phase property if and only if there exists a storage function candidate V(x) *and a function* η(x, u) *such that the dissipation inequality*

$$\frac{\partial V}{\partial x}(f(x) + G(x)u) < \|F_r(x, u)\|^2 \eta(x, u) \tag{9}$$

is satisfied for all u *and all nonzero* x *and where* F_r *is called the output derivative array defined as follows*

$$F_r(x, u) := \begin{bmatrix} h_1(x) \\ \dot{h}_1(x) \\ \vdots \\ h_1^{(r_1)}(x) \\ h_2(x) \\ \vdots \\ h_p^{(r_p)}(x) \end{bmatrix} \tag{10}$$

with $\dot{y}_i = \dot{h}_i(x) = \frac{\partial h_i}{\partial x}(f(x) + G(x)u)$ for all $1 \leq$ i $\leq$ p.

The above theorem is also valid for nonaffine nonlinear systems, that is, where $\dot{x} = f(x, u)$.[24] To use the above result for large-scale systems study, observe that a centralized model of large-scale systems described by (8) can be obtained by augmenting (1) for different i, $1 \leq i \leq N$. The above statement can then be applied to the resulting model. Such analysis may, however, be difficult when the dimension of the nonlinear system (8) is large (which is often the case in plantwide systems) due to the geometric and computational complexities involved. These, for instance, are due to the nontrivial search for the normal form of large-scale systems, the candidate storage and the corresponding supply functions.

Large-Scale Minimum Phase Analysis

In this section, we introduce an alternative approach to determine the minimum phase property of large-scale systems using the concept of dissipative systems. Instead of

†The relative degree of Σ_i about a point x_i^o is defined as the smallest integer ρ_i such that $\mathcal{L}_{g_i}\mathcal{L}_{f_i}^k h_i(x_i) = 0$, $0 \leq k \leq \rho_i - 2$, and $\mathcal{L}_{g_i}\mathcal{L}_{f_i}^{\rho_i-1} h_i(x_i) \neq 0$ for all xi in a neighborhood of x_i^o, where $\mathcal{L}_X Y$ denotes the Lie derivative of Y with respect to a vector field X. If the property holds for all $x_i \in \mathbb{R}^{n_i}$, then Σ_i is said to have a strong relative degree ρ_i.

using a single dissipation inequality for the overall systems as stated in Theorem 1, a collection of dissipativity inequalities (one for each subsystem) are derived as follows. Consider that the output y_i for each Σ_i is controlled such that in the transformed coordinate, $z_i \equiv 0$. Based on these constrained dynamics, the dissipativity for each of subsystem is characterized. The internal stability of the network is then derived based on the dissipativity of each of its subsystems and its topology. Formally, the main result of this section is stated as follows.

Theorem 2. *Assume that there exists a smooth change of coordinate such that Σ_i can be globally transformed into $\tilde{\Sigma}_i$ for all $1 \leq i \leq N$. The large-scale system Σ has a global minimum phase property if there exist, for each Σ_i, a positive–definite storage function $V_i(\zeta_i)$, a scalar function $\alpha_i(\zeta_i)$, and a vector function $\beta_i(\zeta_i)$ such that the dissipativity inequality*

$$\frac{\partial V_i}{\partial \zeta_i}(\psi_i(0,\zeta_i)+\Psi(0,\zeta_i)\tilde{u}_i) \leq \alpha_i(\zeta_i)+\beta_i(\zeta_i)\tilde{u}_i \tag{11}$$

is satisfied and

$$\sum_{i=1}^{N}\left(\alpha_i(\zeta_i)+\beta_i(\zeta_i)H_{ij}\tilde{h}_j(0,\zeta_j)\right) \leq 0 \tag{12}$$

for all $\tilde{u}$ and $\tilde{y}$.

Proof. The global minimum phase property is satisfied if the interconnection of the zero dynamics is stable. To obtain the zero dynamics for each system, let $z_i \equiv 0$ for each $\tilde{\Sigma}_i$ in (6). Suppose now that the zero dynamics ζ_i of each $\tilde{\Sigma}_i$ satisfy the dissipation inequality (11). As $V_i > 0$ for each $\tilde{\Sigma}_i$, we can build the overall storage function for the large-scale network by summing the storage functions from all subsystems, that is, such that $V=\sum_{i=1}^{N} V_i$. Hence, the derivative of the overall storage function satisfies the dissipation inequality (12), noting the relationship $\tilde{u}_i=H_{ij}\tilde{y}_j$. The stability result then follows directly from standard Lyapunov stability arguments.■

Observe that each subsystem is minimum phase if $\alpha_i(\zeta_i) < 0$ in (11) (the zero dynamics of the system is stable when $\tilde{u}_i=0$). From (12), it is clear that if $\alpha_i < 0$ for all $1 \leq i \leq N$, large-scale minimum phase may not be achieved. Conversely, it also shows that if some subsystems are non-minimum phase, the overall zero dynamics of the large-scale system may still be stable (in the sense of Lyapunov) due to the system interactions. Note that in the latter case, we need to ensure the network is zero-state detectable[33] in the sense that $\tilde{y}_i(x_i(t)) \equiv 0$ implies $x_i(t) \to 0$ as $t \to \infty$. The key here is to simultaneously find the storage function and the supply rate for each Σ_i such that the dissipation inequality (11) and network constraint (12) are satisfied.

Finding a suitable nonlinear storage function and a supply rate that satisfy the dissipation inequality (11) in Theorem 2 is a challenging task in general. In the case where all the vector fields of system (1) are polynomials, it is sufficient to search for a storage and a supply function expressed as polynomials. In this case, the dissipation inequality (11) can be effectively verified using the recent advances in semidefinite programming and sum of squares decomposition. These are relevant to process control problems, because the dynamics of many chemical and biological processes can be well approximated or transformed into polynomial nonlinearities. The results by Magyar et al.,[34] in particular, can be used to transform most state space models of process control systems into Lotka–Volterra series (which consists of polynomial vector fields). In the "Illustrative Example" section, we illustrate the application of Theorem 2 for a class of polynomial nonlinear system.

By further specifying the structure of inequality (11), an ISS property[32] can be obtained. This property assigns a bound of the interaction effects on the internal states. In the context of process control, such property is important if a large deviation of the internal states is not allowed (e.g., if the internal states are related to the safety of the process operation, such as pressure and temperature). A nonlinear system $\dot{x}=f(x,u)$ with $0=f(0,0)$ is said to have the ISS property if there exists a smooth positive definite scalar function $V(x)$ that satisfies

$$\frac{\partial V}{\partial x}f(x,u) \leq -\kappa_1(|x|)+\kappa_2(|u|) \tag{13}$$

where $|\cdot|$ denotes the usual Euclidean norm for vectors, whereas $\kappa_1(\cdot)$ and $\kappa_2(\cdot)$ are class $\mathcal{K}$ functions, that is, κ_l is monotonously increasing with $\kappa_l(0)=0$ for $1 \leq l \leq 2$. The ISS property, defined in this article with respect to the zero dynamics and the interconnecting input, provides an upper bound to the system behavior in terms of both the internal state and the input variables. These bounds may be set based on the desired performance, leading to the following corollary.

Corollary 1. *Assume that, for $i = 1, \ldots, N$, there exists a smooth change of coordinates such that each Σ_i is globally transform to $\tilde{\Sigma}_i$. Then, the zero dynamics of each Σ_i has the ISS property if there exists smooth storage functions $V_i(\zeta_i)$, such that the dissipation inequality (11) is satisfied, and $\mathcal{K}$-class functions κ_i such that*

$$\psi_i(0,\zeta_i)+\kappa_i(|\zeta_i|) \leq -c_i\frac{\partial V_i}{\partial \zeta_i}\|\Psi_i(0,\zeta_i)\|^2, \qquad c_i > 0 \tag{14}$$

Proof. From Theorem 2, we have

$$\dot{V}_i(\zeta_i)=\frac{\partial V_i}{\partial \zeta_i}(\psi_i(0,\zeta_i)+\Psi(0,\zeta_i)\tilde{u}_i) \leq \alpha_i(\zeta_i)+\beta_i(\zeta_i)\tilde{u}_i \tag{15}$$

Using norm inequality and the fact that $2ab \leq a^2 + b^2$

$$\dot{V}_i \leq \alpha_i(\zeta_i)+\sigma_i\|\beta_i(\zeta_i)\|^2+\sigma_i^{-1}\|\tilde{u}_i\|^2 \tag{16}$$

where σ_i is a positive scalar. Therefore, we can find a class $\mathcal{K}$ function $\tilde{\kappa}_i(\cdot)$ such that

$$\tilde{\kappa}_i(|\tilde{u}_i|) \leq \sigma_i^{-1}\|\tilde{u}_i\|^2 \tag{17}$$

Observe that the first two terms on the right-hand side of (16) can be selected such that

$$\begin{aligned}\alpha_i(\zeta_i)+\sigma_i\|\beta_i(\zeta_i)\|^2 &= \frac{\partial V_i}{\partial \zeta_i}\psi_i(0,\zeta_i)+\sigma_i\|\frac{\partial V_i}{\partial \zeta_i}\Psi_i(0,\zeta_i)\|^2 \\ &\leq \frac{\partial V_i}{\partial \zeta_i}\psi_i(0,\zeta_i)+\sigma_i\|\frac{\partial V_i}{\partial \zeta_i}\|^2\|\Psi_i(0,\zeta_i)\|^2\end{aligned} \tag{18}$$

By the condition in the theorem, the ISS property (13) is satisfied.■

For some practical plantwide problems, a global condition can be too strong and difficult to analyze. Where local results are appropriate, the dissipativity analysis detailed earlier can be performed based on the linearized systems.[35] In this case, the global transformation from Σ_i to $\tilde{\Sigma}_i$ can be relaxed into local conditions (which are far weaker—see the results by Byrnes and Isidori[31]). Furthermore, for each Σ_i, it is sufficient to take a quadratic storage $V_i=\zeta_i^T P_i \zeta_i$, $P_i>0$, and supply rate $s_i(\tilde{y}_i,\tilde{u}_i)=\tilde{y}_i^T Q_i \tilde{y}_i+2\tilde{y}_i^T S_i \tilde{u}_i+\tilde{u}_i^T R_i \tilde{u}_i$.[36] This type of dissipativity is often referred to as the (Q, S, R)-dissipativity. Under this paradigm, the internal stability of large-scale system given in Eq. 1 can be evaluated locally by solving a linear matrix inequality (LMI) problems, that can be solved efficiently by semidefinite programming. Suppose for each Σ_i, the linearization of the zero dynamics can be represented as

$$\begin{aligned} \dot{\zeta}_i &= A_i \zeta_i + B_i \tilde{u}_i \\ \tilde{y}_i &= C_i \zeta_i \end{aligned} \tag{19}$$

Theorem 3. *Assume that there exists a smooth change of coordinate such that* Σ_i *can be locally transformed into* $\tilde{\Sigma}_i$ *for all* $1 \le i \le N$. *The large-scale system* Σ *has a local minimum phase property if the system is locally zero-state detectable and the inequality*

$$\begin{bmatrix} I & 0 \\ 0 & I \\ A_i & B_i \\ C_i & 0 \end{bmatrix}^T \begin{bmatrix} 0 & 0 & -P_i & 0 \\ 0 & R_i & 0 & S_i^T \\ -P_i & 0 & 0 & 0 \\ 0 & S_i & 0 & Q_i \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ A_i & B_i \\ C_i & 0 \end{bmatrix} > 0 \tag{20}$$

is satisfied with $P_i=P_i^T>0$ *for* $P_i \in \mathbb{R}^{n_i \times n_i}$, $Q_i \in \mathbb{R}^{p_i \times p_i}$, $S_i \in \mathbb{R}^{p_i \times m_i}$, *and* $R_i \in \mathbb{R}^{m_i \times m_i}$, *under the stability constraint*

$$Q+H^T S^T+SH+H^T RH<0 \tag{21}$$

where $Q=diag(Q_1,\ldots,Q_N)$, $S=diag(S_1,\ldots,S_N)$, *and* $R=diag(R_1,.,R_N)$.

Proof. The proof is motivated from the work by Rojas et al.[18] We provide a sketch of the proof here. Equation 20 is essentially (11) with respect to a linear system and quadratic storage function and supply rate. Equation 21 is equivalent to (12) if each Σ_i is (Q, S, R)-dissipative. Finally, the local statement follows through from the arguments made by Byrnes and Isidori.[31] ■

We would like to briefly remark here that, while Theorem 3 is weaker than Theorem 2, the result is widely applicable for practical problems as the procedure (which only involves linearization and solving a LMI problem) is easy to perform and can be computed reliably. For many real problems, it has been reported that such a linear analysis in most cases provides sufficient confidence, especially in guaranteeing stability in the relevant operating region of systems. A nonlinear control law is usually preferred to improve the performance of the plantwide system. From this perspective, the result of Theorem 3 is valuable if we aim to study the (local) internal stability of a plantwide system. The performance of the controlled variable y_i (or z_i in the transformed coordinate) is handled separately by a nonlinear control law to be discussed in the "Interconnection Decoupling" section.

We conclude this section by making the following statement about the link between the above interaction analysis and the overall stabilizability of large-scale systems.

Corollary 2. *The large-scale system* Σ *is asymptotically stabilized on compacta*[‡] *if the controlled variables can be stabilized to their desired equilibrium values and the interconnections of the zero dynamics of its subsystems are globally asymptotically stable. It is locally asymptotically stable if the controlled variables can be driven to their desired equilibrium values and the interconnections of the zero dynamics of its subsystems are locally asymptotically stable.*

Proof. The first result is a direct consequence of Theorem 7.2 by Byrnes and Isidori.[31] The second result comes from Lemma 4.3 by Byrnes and Isidori.[31] The (local) minimum phase property implied therein translates to the (local) asymptotical stability of the interconnections of the zero dynamics presented in this article. ■

One might expect that global stabilization can be achieved if the controlled variables for each Σ_i can be driven to their nominal values and the interconnections of the zero dynamics of each Σ_i are globally asymptotically stable. This is not always true due to the negative results demonstrated by Sussmann.[37] In fact, a stronger bounded-input-bounded-state condition (from z_i to ζ_i) is required to show global asymptotic stabilization. However, Byrnes and Isidori[31] showed that a weaker stabilizability notion (defined on compacta) can be derived without requiring the strong bounded-input-bounded-state property. For most practical purposes, the stabilization based on this weaker property is sufficient.

The results presented so far on the effects of internal dynamics on the stability of large-scale systems assumed that the controlled variables reached their desired equilibrium values. However, as previously mentioned in the "Introduction," the stability (and stabilizability) properties of the controlled variables are affected by the interconnection variables. In the next section, an approach to study and counter these effects is described, based on the notion of disturbance decoupling.[30]

Interconnection Decoupling

The results thus far are mainly concerned about the effects of the interactions of decentralized zero dynamics on the overall stability of large-scale systems. For the purpose of stabilization of large-scale systems, the conditions that ensure the convergence of the controlled variables to the equilibrium under a feedback control law need to be studied. This can be viewed from the solvability of interconnection decoupling under nonlinear state feedback control law. This is analogous to the disturbance decoupling problem,[25,26] where a control law is designed to eliminate the effects of external disturbances. The main idea here is to eliminate the effects of interactions, by the action of feedback control laws, from the dynamics of the controlled variables and to drive the controlled outputs to their desired values. Figure 1 shows the control structure for each subsystem that is considered in this paper.

We define the interconnection characteristic index following the definition of the disturbance characteristic index by Marino et al.[30] We denote $\tilde{u}_i=\text{col}\left(\tilde{u}_{1,i},\ldots,\tilde{u}_{N,i}\right) \in \mathbb{R}^{m_i}$, $W_i=[\,w_{i,1} \;\cdots\; w_{i,m_i}\,]$.

Definition 2. *The interconnection characteristic index of* Σ_i *is defined as the smallest integer* ν_i *such that*

[‡]*This is a weaker notion of stabilization introduced in Ref. 31. About an equilibrium x*, a system is said to be stabilizable on compacta if for any compact set of initial states* Ω*, there exists a smooth state feedback law (which may depend on* Ω*) such that the closed-loop system is locally asymptotically stable about x* and* Ω *is contained in the domain of attraction of x*.*

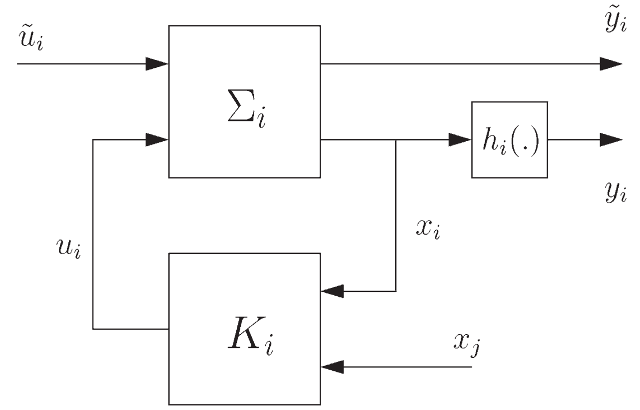

Figure 1. Closed-loop configuration for each subsystem (K_i denotes the controller).

$$\begin{array}{ll}\mathcal{L}_{w_{il}}\mathcal{L}_{f_i}^{k}h_i(x_i)=0, \quad 1\leq l\leq m_i, \quad 0\leq k\leq \nu_i-2 \\ \mathcal{L}_{w_{il}}\mathcal{L}_{f_i}^{\nu_i-1}h_i(x_i)\neq 0, \quad \text{for some } l, \quad 1\leq l\leq m_i\end{array} \tag{22}$$

Observe that the above definition is very similar to the definition of relative degree of a nonlinear system. The relative degree is defined based on the controlled variables-manipulated variables pair. The interconnection characteristic index, on the other hand, is defined based on the controlled variables-interconnecting input variables pair.

For notational purposes, the representation of large-scale systems using graph theory is considered (see the article by Jillson and Ydstie[38] for an example of graph representations of process control system networks). The following definitions are used in the sequel.

Definition 3. *A directed graph (or digraph) is a pair* (V, E)*, where $V=\{v_1,\cdots,v_N\}$ is a finite set of vertices and* E*, a subset of* V × V*, is a finite set of edges. If $(v_i,v_j)\in E$, an edge is said to exist from* v_i *to* v_j*. Given a digraph* (V, E)*, the following can be defined:*

- *A path of length* k *from $v_i\in V$ to $v_j\in V$ is an ordered set $\{v_{i0},\cdots,v_{ik}\}$ with $v_{i0}=v_i$ and $v_{ik}=v_j$, such that $(v_{il},v_{il+1})\in E$ for all $0\leq l\leq k-1$;*
- *A vertex $v_j\in V$ is said to be* q*-reachable from $v_i\in V$ (denoted as v_j R_q v_i) if there exists a path of length of* q *from* v_i *to* v_j.

Consider the digraph (V, E) of large-scale system Σ with $v_i=\Sigma_i$. Define the q upstream neighborhood of Σ_i as $\Pi_{i(q)}=\{\Sigma_j|\Sigma_i R_q \Sigma_j \forall \Sigma_j\in\Sigma\}$ and $\Pi_{i(0)}=\emptyset$. In the sequel, we refer by $x_{i(q)}$ as the states of all $\Sigma_j\in\Pi_{i(q)}$. Figure 2 illustrates the idea of upstream neighborhood. Under the above definitions, the interconnecting input $\tilde{u}_i$ is denoted by $\tilde{u}_i(z_{i(1)},\zeta_{i(1)})$. This means that the interconnecting input to a subsystem is a function of the states of the neighboring systems. In process systems, $\tilde{u}_i$ may correspond to the convective mass or energy flow to Σ_i.[18]

Theorem 4. *Assume that there exists a smooth change of coordinates such that* Σ_i *can be globally (locally) transformed into $\tilde{\Sigma}_i$ for all* $1\leq i\leq N$*. In addition, suppose that for each* Σ_i, Φ_{ik} *in (6) has the triangular structure*

$$\Phi_{ik}(z_i)=\Phi_{ik}(z_{i,1},\ldots,z_{i,k},\zeta_i), \quad 1\leq k\leq \rho_i \tag{23}$$

Then, the interconnection decoupling problem is (locally) solvable for each subsystem under distributed control laws if:

1. *the large-scale interconnections of the zero dynamics are stable in the sense of Theorem 2;*
2. *the feedback controller for each subsystem has access to the state measurements of* Σ_i *and, if* $\rho_i\geq \nu_i$*, its upstream neighborhood systems* $\Pi_{i(q)}$ *for all* $1\leq q\leq \rho_i-\nu_i+1$*;*
3. *the feedback controller for each subsystem has access to the neighboring control laws $\left(u_{i(q)}^{(\rho_i-\nu_i-q)},\ldots,u_{i(q)}\right)$ that are independent of* u_i*, for all* $0\leq q\leq \rho_i-\nu_i$*; and*
4. *the recycle paths (from* Σ_i *to* Σ_i*) satisfy* Σ_i R_q Σ_i *with* $q=0$ *or* $q>\rho_i-\nu_i$.

Proof. The proof is divided into two cases.

Case 1: $\rho_i<\nu_i$. This is a sufficient and necessary condition for exact decoupling.[28] In this case, the interaction effect from the interconnection can be completely decoupled from the controlled output and its derivatives ($\Phi_i=0$ in (6)) by decentralized control action (5) (e.g., by designing v_i as a feedback linearization control law).

Case 2: $\rho_i\geq\nu_i$. Given that (23) is satisfied, the interconnection decoupling control law can be constructed recursively for each Σ_i, $1\leq i\leq N$. First, note the following facts:

- the global transformation of (1) into (6) requires both $(\rho_i,\ \nu_i)\geq(1,1)$,
- from Definition 2, $\Phi_{i,k}\neq 0$, $1\leq k\leq\rho_i$, if $\nu_i\leq k$ and $\Phi_{i,k}=0$ otherwise,
- the higher derivative of the interconnecting input $\tilde{u}_i$ satisfies

$$\tilde{u}_i^{(r)}=\lambda\left(z_{i(r+1)},\zeta_{i(r+1)},z_{i(q)},\zeta_{i(q)},u_{i(q)}^{(r-q)},\ldots,u_{i(q)}\right),\ 1\leq q\leq r$$

Choose $V_{i,1}=\frac{1}{2}z_{i1}^2$. Then

$$\frac{dV_{i,1}}{dt}=z_{i1}\left(z_{i,2}+\Phi_{i,1}\tilde{u}_i\right) \tag{24}$$

If the conditions in the theorem are satisfied, it is possible to define

$$z_{i,2}^{*}=-\gamma_{i,1}z_{i,1}-\Phi_{i,1}\tilde{u}_i \tag{25}$$

with $\gamma_{i,1}>0$ such that by equating $z_{i,2}=z_{i,2}^{*}$, we have

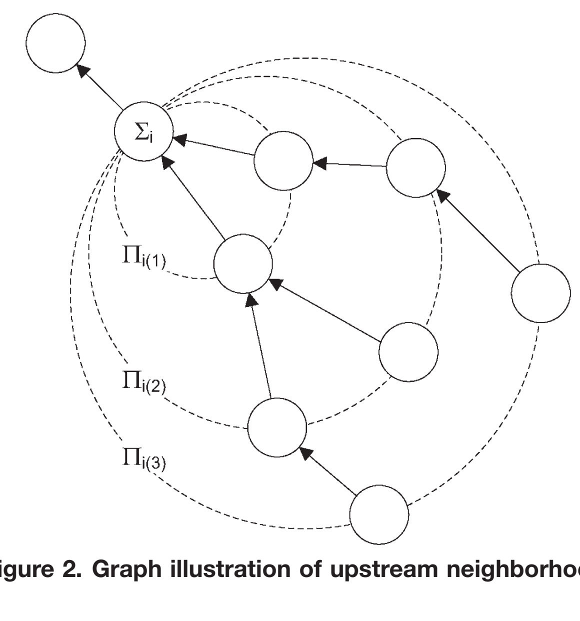

Figure 2. Graph illustration of upstream neighborhood.

$$\frac{dV_{i,1}}{dt}=-\gamma_{i,1}z_{i,1}^2<0 \tag{26}$$

for all $z_{i,1}\neq 0$ and $V_{i,1}=0$ only when $z_{i,1}=0$. Note that if $\rho_i=1$, $v_i=1$ and by choosing $v_i=z_{i,2}^*$, the control law stabilizes the output. Obviously, in this case

$$z_{i,2}^*=a\left(z_{i(1)},\zeta_{i(1)},z_i,\zeta_i\right) \tag{27}$$

concluding the proof of the theorem when $\rho_i=1$.

We now consider the case where $\rho_i>1$. Denote $\tilde{\Phi}_{i,k}=\Phi_{i,k}\tilde{u}_i$. Using the facts listed in the beginning of the proof, we have, for $1\leq q\leq r$

$$\tilde{\Phi}_{i,k}^{(r)}=\begin{cases}0 & \text{if } k<v_i\\ b\left(z_{i(r+1)},\zeta_{i(r+1)},z_{i(q)},\zeta_{i(q)},u_{i(q)}^{(r-q)},..,u_{i(q)},z_i,\zeta_i\right), & \text{if } k\geq v_i,\rho_i\geq r+k\end{cases} \tag{28}$$

Recall that by the chain rule, we have

$$c_1^{(1)}(x)=c_2(x,x^{(1)}) \tag{29}$$

Using (6), (28), and (29), we see that

$$z_{i,2}^*=\sigma_{i,1}\left(z_{i,1},\tilde{\Phi}_{i,1}\right) \tag{30}$$

and

$$\frac{dz_{i,2}^*}{dt}=\sigma_{i,1}'\left(z_{i,1},z_{i,2},\tilde{\Phi}_{i,1},\tilde{\Phi}_{i,1}^{(1)}\right) \tag{31}$$

Define $\tilde{z}_{i,2}=z_{i,2}-z_{i,2}^*$. Then, taking $V_{i,2}=V_{i,1}+\frac{1}{2}\tilde{z}_{i,2}^2$, we have

$$\frac{dV_{i,2}}{dt}=\tilde{z}_{i,2}\left(z_{i,3}+\tilde{\Phi}_{i,2}+\sigma_{i,1}'\right)+z_{i,1}\tilde{z}_{i,2}-\gamma_{i,1}z_{i,1}^2 \tag{32}$$

Choosing

$$z_{i,3}^*=-\gamma_{i,2}\tilde{z}_{i,2}-\tilde{\Phi}_{i,2}-\sigma_{i,1}'-z_{i,1} \tag{33}$$

with $\gamma_{i,2}>0$ and letting $z_{i,3}=z_{i,3}^*$, we have

$$\frac{dV_{i,2}}{dt}=-\gamma_{i,1}z_{i,1}^2-\gamma_{i,2}\tilde{z}_{i,2}^2<0 \tag{34}$$

for all $\left(z_{i,1},\tilde{z}_{i,2}\right)\neq 0$ and $V_{i,2}=0$ only when $z_{i,1}=\tilde{z}_{i,2}=0$. Observe that

$$z_{i,3}^*=\sigma_{i,2}\left(z_{i,1},z_{i,2},\tilde{\Phi}_{i,1},\tilde{\Phi}_{i,1}^{(1)},\tilde{\Phi}_{i,2}\right) \tag{35}$$

Therefore

$$\frac{dz_{i,3}^*}{dt}=\sigma_{i,2}'\left(z_{i,1},z_{i,2},z_{i,3},\tilde{\Phi}_{i,1},\tilde{\Phi}_{i,1}^{(1)},\tilde{\Phi}_{i,1}^{(2)},\tilde{\Phi}_{i,2},\tilde{\Phi}_{i,2}^{(1)}\right) \tag{36}$$

Performing this procedure recursively, it can be shown that for all $3\leq p\leq \rho_i$, choosing

$$V_{i,p-1}=\sum_{m=1}^{p-1}\frac{1}{2}\tilde{z}_{i,m}^2 \tag{37}$$

where $\tilde{z}_{i,1}=z_{i,1}$ and $\tilde{z}_{i,m}=z_{i,m}-z_{i,m}^*$ for $1<m\leq p$ and taking

$$z_{i,p}^*=-\gamma_{i,p-1}\tilde{z}_{i,p-1}-\tilde{\Phi}_{i,p-1}-\sigma_{i,p-2}'-z_{i,p-2} \tag{38}$$

where $\sigma_i',m=z_{i,m+1}^{*(1)}$ and $z_{i,2}^*$ is as in (25), we have

$$\frac{dV_{i,p-1}}{dt}=-\sum_{m=1}^{p-1}\gamma_{i,m}\tilde{z}_{i,m}^2<0 \tag{39}$$

for all $\left(\tilde{z}_{i,1},\ldots,\tilde{z}_{i,p-1}\right)\neq 0$ and $V_{i,p-1}=0$ only when $\tilde{z}_{i,1}=\cdots=\tilde{z}_{i,p-1}=0$. In addition

$$z_{i,p}^*=\sigma_{i,p-1}\left(z_{i,1},\ldots,z_{i,p-1},\tilde{\Phi}_{i,1},\ldots,\tilde{\Phi}_{i,1}^{(p-2)},\ldots,\tilde{\Phi}_{i,k},\ldots,\tilde{\Phi}_{i,k}^{(p-k-1)},\ldots,\tilde{\Phi}_{i,p-1}\right) \tag{40}$$

Finally, substituting $p=\rho_i$ and defining $V_{i,\rho_i}=V_{i,\rho_i-1}+\frac{1}{2}\tilde{z}_{i,\rho_i}^2$, we have

$$\frac{dV_{i,\rho_i}}{dt}=\tilde{z}_{i,\rho_i}\left(v_i+\tilde{\Phi}_{i,\rho_i}+\sigma_{i,p-1}'\right)+z_{i,\rho_i-1}\tilde{z}_{i,\rho_i}-\sum_{m=1}^{\rho_i-1}\gamma_{i,m}\tilde{z}_{i,m}^2 \tag{41}$$

Taking

$$v_i=-\gamma_{i,\rho_i}\tilde{z}_{i,\rho_i}-\tilde{\Phi}_{i,\rho_i}-\sigma_{i,\rho i}'-z_{i,\rho_i-1} \tag{42}$$

we obtain

$$\frac{dV_{i,\rho_i}}{dt}=-\sum_{m=1}^{\rho_i}\gamma_{i,m}\tilde{z}_{i,m}^2<0 \tag{43}$$

and the control law stabilizes the output to its equilibrium. By defining $\sigma\prime_{i,\rho_i-1}=z_{i,\rho}^{*(1)}$, we conclude that

$$v_i=\sigma_{i,\rho_i}\left(z_{i,1},\ldots,z_{i,\rho_i},\tilde{\Phi}_{i,1},\ldots,\tilde{\Phi}_{i,1}^{(\rho_i-1)},\ldots,\tilde{\Phi}_{i,k},\ldots,\tilde{\Phi}_{i,k}^{(\rho_i-k)},\ldots,\tilde{\Phi}_{i,\rho_i}\right) \tag{44}$$

By (28), the proof is complete.■

Remark 1. *The geometric conditions that guarantee (23) can be found in the article by Marino et al.[30] Such conditions impose some limitations on the application of the proposed results to general process systems. However, the restrictions can be mild, as in most practical process control, the controlled and manipulated variables are usually selected such that the system has a relative degree of one.[39] In such case, (23) is trivially satisfied. For the case where the structural condition (23) is not satisfied, hierarchical control structure plays an important role for the selection of the manipulated variables, especially when there are constraints on the relative degree assignment with respect to the desired controlled variables. In process control, for example, hierarchical procedure is often used to assign the set point of a controlled variable (e.g., an extensive quantity such as inventory) as the manipulated variable for another desired controlled variable (e.g., the desired intensive quantity such as product composition). See the article by Skogestad[3] for more details. In the case where such selection is not possible for some units even with the hierarchical strategy, we may fix a simple control loop and include the dynamics of those units into the internal dynamics of the overall network. See the first example in the "Illustrative Example" section that illustrates this effect to some extent.*

Remark 2. *The conditions (1)–(4) in Theorem 4 should not be viewed as limitations of the technique, but rather as guides to make a proper selection of manipulated/controlled variables pair. That is, if the control design objective is to accomplish interconnection decoupling, and, then, the control structure has to be adjusted such that (1) is satisfied and the relative degree and the interconnection characteristic indices satisfies (2)–(4).*

The results of Theorem 4 imply that measurements of higher derivatives of the control laws for the neighboring units may be required to achieve the interconnection decoupling in Σ_i. In some cases, these derivatives are computable by the chain rules (29), for example, if the neighboring control laws are state dependent. By substituting the state functions of the neighboring control laws (and their derivatives) into the control action (44) through (28), it can be seen that interconnection decoupling control law for Σ_i is dependent on the states of a large subset of the overall network. In fact, it is not difficult to see that the more information the control law $u_{i(q)}$ in (28) involves, the larger is the subset of the network that needs to be considered for the computation of (44). At least from the perspective of control algorithm complexity, low information distribution is desirable. Observe that the amount of required information is also related to the relative degrees and interconnection characteristic indices of each subsystem. By lowering the relative degrees and increasing the interconnection indices, the amount of required distribution can be lowered. As shown in the next Corollary, in the case where $\rho_i = v_i$, the interconnection decoupling law for Σ_i only requires the neighboring information from $\Pi_{i(1)}$.

Corollary 3. *Assume that there exists a smooth change of coordinate such that Σ_i can be globally transformed into $\tilde{\Sigma}_i$ for all $1 \le i \le N$. In addition, suppose that for each Σ_i, Φ_{ik} in (6) has the triangular structure*

$$\Phi_{ik}(z_i) = \Phi_{ik}\left(z_{i,1}, \ldots, z_{i,k}, \zeta_i\right), \quad 1 \le k \le \rho_i \tag{45}$$

If the large-scale interconnection of the zero dynamics is stable and $\rho_i = v_i$, the interconnection decoupling problem is solvable for each subsystem under distributed control laws that have the access of the states in Σ_i and $\Pi_{i(1)}$.

The proof to the above corollary is straightforward from the proof to Theorem 4. In this case, the topology of the signal transfers resembles that of the physical process network and the resulting distributed control law can be related to a feedback/feedforward controller form.[25,26]

If each Σ_i can be globally transformed into $\tilde{\Sigma}_i$, the minimum phase property of the network can be studied using the results presented in the "Large-Scale Minimum Phase Analysis" section. In view of the possible distributed control law (for interconnection decoupling), we show that a more flexible structure of the internal dynamics than that introduced in (6) is permitted for the large-scale minimum phase analysis based on dissipativity if $\rho_i \le v_i$. That is, instead of (6), suppose Σ_i can be globally transformed into

$$\hat{\Sigma}_i : \begin{cases} \dot{z}_{i,1} & = z_{i,2} + \Phi_{i,1}(z_i)\tilde{u}_i \\ & \vdots \\ \dot{z}_{i,\rho_i - 1} & = z_{i,\rho_i} + \Phi_{i,\rho_i - 1}(z_i)\tilde{u}_i \\ \dot{z}_{i,\rho_i} & = v_i + \Phi_{i,\rho_i}(z_i)\tilde{u}_i \\ \dot{\zeta}_i & = \psi_i(z_i, \zeta_i, v_i) + \Psi_i(z_i, \zeta_i)\tilde{u}_i \\ y_i & = z_{i,1} \end{cases} \tag{46}$$

This structure corresponds to a process system where the manipulated variables also enter the zero dynamics (e.g., in distillation column as shown in the "Illustrative Example" section). In this case, we need to ensure one more condition, that is, for each $\hat{\Sigma}_i$

$$\psi_i(0, \zeta_i, v_i) = \psi_{i,1}(\zeta_i) + \psi_{i,2}(\zeta_i)\tilde{u}_i \tag{47}$$

or in the original coordinates, it means that we have

$$u_i = \sigma_{i,1}(x_i) + \sigma_{i,2}(x_i)\tilde{u}_i \tag{48}$$

Where the above is true, we see that both Theorems 2 and 3 can still be applied.

Illustrative Examples

To illustrate the main results developed in the sections of "Large-Scale Minimum Phase Analysis" and "Interconnection Decoupling," we consider two examples motivated from real process networks. The first example is derived from the biological network studied by Laub and Loomis.[40] We show how the global minimum phase property can be studied in this case. The second is a standard example of plantwide system involving a reactor–separator network.[19] We demonstrate how local minimum phase property can be analyzed. In both examples, the interconnection decoupling control law is implemented.

Biological network

The original problem studies the oscillatory behavior of adenosine 3,5-cyclic monophosphate in homogenous populations of Dictyostelium cells. In this example, the nonlinearities in the vector fields are modified to better illustrate the proposed methodology. A few manipulated variables and internal dynamics are also introduced. The modified network is viewed as a network of seven subsystems. The model is described below.

$$\begin{aligned}
&\Sigma_1 : \dot{x}_1 = -k_2\left(x_1^3 + x_1^2 + x_1\right) + k_1\tilde{u}_1 \\
&\Sigma_2 : \dot{x}_2 = -k_4 x_2 + k_3\tilde{u}_2 \\
&\Sigma_3 : \begin{cases} \dot{x}_3 & = \begin{bmatrix} -k_6 x_{3,1} \\ -x_{3,1} + x_{3,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_3 + \begin{bmatrix} -k_6 x_{3,2} & k_5 \\ 0 & x_{3,1} \end{bmatrix} \tilde{u}_3 \\ y_3 & = x_{3,2} \end{cases} \\
&\Sigma_4 : \dot{x}_4 = -k_8 x_4^3 + k_7 x_4 \tilde{u}_4 \\
&\Sigma_5 : \begin{cases} \dot{x}_5 & = \begin{bmatrix} -k_{10}\left(x_{5,1} + x_{5,2}^2\right) \\ \alpha\left(x_{5,1}, x_{5,2}\right) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_5 + \begin{bmatrix} k_9 & -k_{10} x_{5,2} \\ x_{5,1} x_{5,2} & x_{5,2} \end{bmatrix} \tilde{u}_5 \\ y_5 & = x_{5,2} \end{cases} \\
&\Sigma_6 : \dot{x}_6 = -k_{12} x_6 + k_{11}\tilde{u}_6 \\
&\Sigma_7 : \begin{cases} \dot{x}_7 & = \begin{bmatrix} -k_{14}\left(x_{7,1}^3 + x_{7,1}\right) + x_{7,2} \\ x_{7,1} x_{7,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_7 + \begin{bmatrix} -k_{14} x_{7,1} & k_{13} \\ 1 & 1 \end{bmatrix} \tilde{u}_7 \\ y_7 & = x_{7,2} \end{cases}
\end{aligned} \tag{49}$$

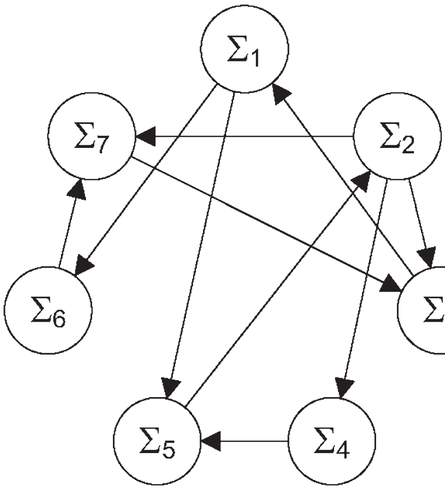

Figure 3. Graph representation of biological network.

where $\tilde{y}_i = x_i$ for $i = \{1, 2, 4, 6\}$ and $\tilde{y}_i = x_{i,1}$ for $i = \{3, 5, 7\}$. The function $\alpha(x_{1,1}, x_{2,2})$ in Σ_5 is assumed to be continuously differentiable in $x_{5,1}$ and $x_{5,2}$ with $\alpha(0, 0) = 0$. Consider $\tilde{u} = H\tilde{y}$, with associated graph representation given in Figure 3, such that

$$H = \left[\begin{array}{ccccccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \tag{50}$$

The objective is to study the interaction effects at the wash-out type equilibrium (i.e., at the origin). The stability of the zero dynamics of network (49) is guaranteed if the conditions in Theorem 2 is satisfied. Observe that there is no manipulated variable available for subsystems 1, 2, 4, and 6 (i.e., they are open-loop). From the overall network perspective, hence, the states of these subsystems also constitute the zero dynamics of the network—their dynamics will determine the internal stability of the plantwide system. Using the sum of squares decomposition tool in SOSTOOL,[41] we compute the following quadratic storage functions

$$\begin{aligned} &V_1 = 4.23\zeta_1^2, \quad V_2 = 2.84\zeta_2^2, \quad V_3 = 11.13\zeta_3^2, \quad V_4 = 2.54\zeta_4^2, \\ &V_5 = 6.49\zeta_5^2, \quad V_6 = 3.46\zeta_6^2, \quad \text{and} \quad V_7 = 4.06\zeta_7^2 \end{aligned} \tag{51}$$

By Theorem 2, we can conclude that the zero dynamics of the overall network is stable.

Observe that in the above example, $\rho_i = v_i = 1$, $i = \{3, 5, 7\}$. By Theorem 4 and the graph representation in Figure 3, it can be shown that if the manipulated variable is chosen such that

$$u_3 = \sigma_3(x_2, x_3, x_7), \; u_5 = \sigma_5(x_1, x_4, x_5), \; \text{and} \; u_7 = \sigma_7(x_2, x_6, x_7) \tag{52}$$

interconnection decoupling for Σ_i, $i = \{3, 5, 7\}$, can be accomplished. In fact, it is easy to show by feedback equivalence to passive systems[42] that assigning

$$\begin{aligned} u_3 &= x_{3,1} - x_{3,2} - x_{3,1}x_{7,1} - \gamma_3 x_{3,2} \\ u_5 &= -\alpha(x_{5,1}, x_{5,2}) - x_{5,1}x_{5,2}x_1 - x_{5,2}x_4 - \gamma_5 x_{5,2} \\ u_7 &= -x_{7,1}x_{7,2} - x_2 - x_6 - \gamma_7 x_{4,1} \end{aligned} \tag{53}$$

with $\gamma_i > 0$, $i = \{3, 5, 7\}$, leads to interconnection decoupling and global stabilization of the output. The amount of information required by each controller is no more than that derived from the analysis.

Consider the case where $y_7 = x_{7,1}$. As a result, we have $\rho_7 = 2$ and $v_7 = 1$. Based on Theorem 4 and the graph representation Figure 3, the control law

$$u_7 = \sigma_7(x_1, x_2, x_5, x_6, x_7) \tag{54}$$

is sufficient for interconnection decoupling in Σ_7. For example, using an exact backstepping approach, the following control law can be designed

$$\begin{aligned} u_7 = &-\gamma_{7,1}k_{13}x_6 + \gamma_{7,1}k_{14}x_{7,1} - k_{13}k_{11}x_1 + \gamma_{7,1}k_{14}x_{7,1}^3 + k_{13}k_{12}x_6 \\ &- k_{14}^2x_2^2x_{7,1} + k_{14}x_2x_{7,2} + k_{14}k_{13}x_6 - 2k_{14}^2x_{7,1}x_2 - 4k_{14}^2x_{7,1}^3x_2 \\ &+ 3k_{14}x_{7,1}^2x_{7,2} - 3k_{14}^2x_{7,1}^5 - 4k_{14}^2x_{7,1}^3 - k_{14}^2x_{7,1} + k_{14}x_{7,2} - \gamma_{7,1}x_{7,2} \\ &+ k_{14}x_{7,1}k_3x_5 - x_{7,1}x_{7,2} + 3k_{14}x_{7,1}^2k_{13}x_6 - x_2 - x_6 - x_{7,1} \\ &+ \gamma_{7,1}k_{14}x_{7,1}x_2 + k_{14}x_2k_{13}x_6 - k_{14}x_{7,1}k_4x_2 \\ &- \gamma_{7,2}\left(x_{7,2} - k_{14}\left(x_{7,1}^2 + x_{7,1}\right) - k_{14}x_{7,1}x_2 + k_{13}x_6 + \gamma_{7,1}x_{7,1}\right) \end{aligned} \tag{55}$$

with $\gamma_{7,1} > 0$ and $\gamma_{7,2} > 0$. Observe that the above law satisfies (54).

The above example also illustrates how the interaction effects between subsystems become increasingly complex with the increasing gap between the relative degree and the interconnection characteristic index. At least from the perspective of interactions, it is desirable to keep the difference between the relative degree and the interconnection index as low as possible.

Reactor–separator network

In this example, we study a network of system consisting of a continuous stirred tank reactor (CSTR) and a distillation column as illustrated in Figure 4. The reactor partially converts reactant A to product B via a first-order exothermic reaction. The yield of the reactor is fed to the distillation column with seven stages. The bottom product of the column mainly consists of B, whereas the top product is recycled back to the reactor. To avoid unnecessary complications, only the material balance is considered in the network. Consequently, it is assumed that appropriate temperature control loops are available to adjust the temperature in both systems.

It is assumed that the reactor is liquid phase and perfectly mixed. In the distillation columns, the following assumptions are made: (1) binary mixture, (2) constant pressure, (3) constant relative volatility α, (4) constant molar flows, (5) no vapor holdup, (6) constant liquid holdup on all trays (except for the condenser), and (7) vapor–liquid equilibrium and

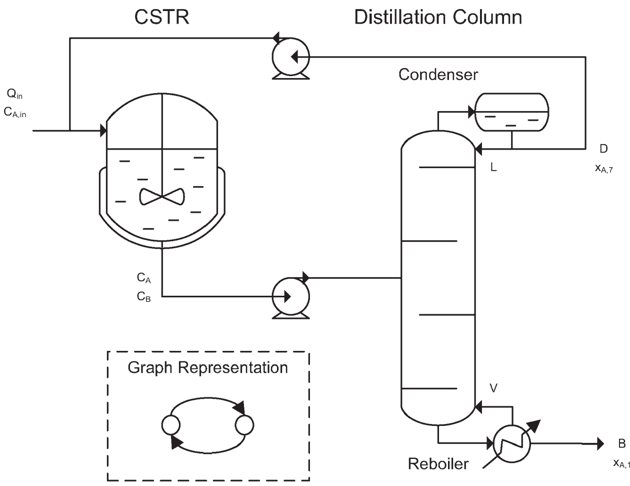

Figure 4. Network of CSTR and distillation column.

perfect mixing on all stages. It is further assumed that the reactor and condenser level is controlled by linear proportional controllers. The parameters are summarized in Table 1. The nominal operating points of the distillation column are provided in Figure 2. With all the above notes, the models of the CSTR and distillation column can be proposed as follows.

Modeling equations for CSTR[43]:

$$\frac{dC_A}{dt}=\frac{Q_{in}}{V_R}C_{A,in}+\frac{D}{V_R}x_{A,7}-\frac{Q}{V_R}C_A-k_0e^{\left(-\frac{E_a}{R_gT}\right)}C_A-\frac{C_A}{V_R}\times\left(Q_{in}+\frac{DM_r}{\rho}-Q\right)$$

$$\frac{dC_B}{dt}=\frac{D}{V_R}\left(1-x_{A,7}\right)-\frac{Q}{V_R}C_B+k_0e^{\left(-\frac{E_a}{R_gT}\right)}C_A-\frac{C_B}{V_R}\times\left(Q_{in}+\frac{DM_r}{\rho}-Q\right)\quad\frac{dV_R}{dt}=Q_{in}+\frac{DM_r}{\rho}-Q$$

where $Q=Q^*+k_c\left(V_R-V_R^*\right)$.

Modeling equations for distillation column[44]:

$$\frac{dx_{A,1}}{dt}=\frac{(L+F)}{N_1}\left(x_{A,2}-x_{A,1}\right)+\frac{V}{N_1}\left(x_{A,1}-y_{A,1}\right)$$

$$\frac{dx_{A,2}}{dt}=\frac{(L+F)}{N_2}\left(x_{A,3}-x_{A,2}\right)+\frac{V}{N_2}\left(y_{A,1}-y_{A,2}\right)$$

$$\frac{dx_{A,3}}{dt}=\frac{(L+F)}{N_3}\left(x_{A,4}-x_{A,3}\right)+\frac{V}{N_3}\left(y_{A,2}-y_{A,3}\right)$$

$$\frac{dx_{A,4}}{dt}=\frac{L}{N_4}\left(x_{A,5}-x_{A,4}\right)+\frac{V}{N_4}\left(y_{A,3}-y_{A,4}\right)+\frac{F}{N_4}\left(x_{A,F}-x_{A,4}\right)$$

$$\frac{dx_{A,5}}{dt}=\frac{L}{N_5}\left(x_{A,6}-x_{A,5}\right)+\frac{V}{N_5}\left(y_{A,4}-y_{A,5}\right)$$

$$\frac{dx_{A,6}}{dt}=\frac{L}{N_6}\left(x_{A,7}-x_{A,6}\right)+\frac{V}{N_6}\left(y_{A,5}-y_{A,6}\right)$$

$$\frac{dx_{A,7}}{dt}=\frac{V}{N_7}\left(y_{A,6}-x_{A,7}\right)-\frac{x_{A,7}}{N_7}(V-L-D)$$

$$\frac{dN_7}{dt}=V-L-D$$

where

$$F=Q(C_A+C_B)$$

$$x_{A,F}=\frac{C_A}{C_A+C_B}$$

$$y_{A,i}=\mathcal{Y}\left(x_{A,i}\right)=\frac{\alpha x_{A,i}}{1+(\alpha-1)x_{A,i}}$$

$$D=D^*+k_c\left(N_7-N_7^*\right)$$

Denote Σ_1 as the CSTR and Σ_2 the distillation column. We have

$$\begin{bmatrix}x_{1,1}\\x_{1,2}\\x_{1,3}\end{bmatrix}=\begin{bmatrix}C_A\\C_B\\V_R\end{bmatrix}\quad\text{and}\quad\begin{bmatrix}x_{2,1}\\\vdots\\x_{2,7}\\x_{2,8}\end{bmatrix}=\begin{bmatrix}x_{A,1}\\\vdots\\x_{A,7}\\N_7\end{bmatrix}$$

Table 1. System Parameters

Parameter	Description	Unit	Value
ρ	Density of A and B	kg/m^3	500
M_r	Molar mass of A and B	kg/kg mol	20
C_A	Concentration of A	kg mol/m^3	3.25[a]
C_B	Concentration of B	kg mol/m^3	9.25[a]
$C_{A,in}$	Inlet concentration of A	kg mol/m^3	10[a]
T	Temperature	K	325
Q_{in}	Reactor inlet flow rate	m^3/h	0.2[a]
Q	Reactor outlet flow rate	m^3/h	0.24[a]
V_R	Volume of reactor	m^3	1
E_a	Activation energy	kcal/kg mol	11,843
k_0	Pre-exponential factor	h^{-1}	5.337×10^7
R_g	Ideal gas constant	kcal/K kg mol	1.987
D	Distillate flow rate	kg mol/h	1.0[a]
F	Distillation feed flow rate	kg mol/h	3.0[a]
L	Liquid flow rate	kg mol/h	8.0[a]
V	Vapor flow rate	kg mol/h	9.0[a]
α	Relative volatility	–	2.0
N_i	Holdup at stage i	kg mol	(see Table 2)
$x_{A,F}$	Feed liquid mole fraction of A	–	(see Table 2)
$x_{A,i}$	Liquid mole fraction of A at stage i	–	(see Table 2)
$y_{A,i}$	Vapor mole fraction of A at stage i	–	(see Table 2)

[a]Evaluated at the operating points.

For this particular case study, we choose

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_A \\ x_{A,1} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} C_{A,in} \\ V \end{bmatrix}$$

We also choose

$$\begin{bmatrix} \tilde{y}_{1,1} \\ \tilde{y}_{1,2} \end{bmatrix} = \begin{bmatrix} F \\ Fx_{A,F} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \tilde{y}_{2,1} \\ \tilde{y}_{2,2} \end{bmatrix} = \begin{bmatrix} D \\ Dx_{A,7} \end{bmatrix}$$

and we have a feedback interconnection topology

$$\begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix}$$

We assume the distillate composition x_7 is already controlled by a proportional controller through the reflux flow rate L, such that $L = L(x_{2,7})$. Note that this is only done to ensure that each subsystem can be represented by SISO structure as in (1). Alternatively, one can design multiple geometric control law for Σ_2.[45] It should be noted that, with $x_{A,i}$, $1 \le i \le 7$ being the liquid composition in the distillation tray, the physical law, given the value of α, ensures that $x_{A,i} < x_{A,i+1}$ for all $1 \le i \le 6$. The system always stays in the positive orthant, and the equilibrium is located away from the origin. Elementary coordinate transformation (using deviation variables) can, however, be applied such that the assumptions on the vector fields considered in this article are still satisfied.

Observe that for both Σ_1 and Σ_2, $\rho_i = 1$ and $\nu_i = 1$. Therefore, based on Theorem 4, we deduce that if the control law has the access to the states in each subsystem and its neighbor, the outputs can be asymptotically stabilized into the desired equilibrium. For example, by feedback equivalence to passive systems approach,[42] the following control law may be found to asymptotically stabilize the output

$$u_1 = \frac{x_{1,3}}{Q_{in}} \left(-\frac{\tilde{u}_{1,2}}{x_{1,3}} + \frac{Q(x_{1,3})}{x_{1,3}} x_{1,1} + k_0 e^{\left(-\frac{E_a}{R_g T}\right)} x_{1,1} + \frac{x_{1,1}}{x_{1,3}} \left(Q_{in} + \frac{M_r}{\rho} \tilde{u}_{1,1} - Q(x_{1,3}) \right) + x^*_{1,1} - x_{1,1} \right)$$

$$u_2 = \frac{L(x_{2,7}) + \tilde{u}_{2,1})(x_{2,1} - x_{2,2}) + N_1 \left(x^*_{2,1} - x_{2,1} \right)}{N_1 \left(x_{2,1} - \mathcal{Y}(x_{2,1}) \right)}.$$

Observe that the zero dynamics of Σ_i are still affine in $\tilde{u}_i$, $i = 1, 2$. Now it is important to check the internal stability of the overall network. Here, we rely on the local stability analysis as formalized in Theorem 3. After performing the linearization on the zero dynamics of both subsystems, the associated LMI problem in Theorem 3 yields

$$Q_1 = \begin{bmatrix} -0.1590 & 0.4413 \\ 0.4413 & -1.9382 \end{bmatrix}, \quad S_1 = \begin{bmatrix} 0.1026 & 0.0038 \\ -0.2373 & -0.2094 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 0.5479 & -0.7311 \\ -0.7311 & 1.2489 \end{bmatrix}$$
$$Q_2 = \begin{bmatrix} -0.9914 & 1.1218 \\ 1.1218 & -1.9131 \end{bmatrix}, \quad S_2 = \begin{bmatrix} -0.0364 & 0.2124 \\ -0.0361 & -0.3141 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.0844 & -0.2241 \\ -0.2241 & 1.0133 \end{bmatrix}$$

One can easily check that both systems are locally zero state detectable and, thus, the network is locally asymptotically stable. The above study shows that the overall network system is locally stabilizable under the control law u_1 and u_2. Figure 5 shows the trajectories of the closed-loop systems in response to 10 and 5% pulse disturbances in Q_{in} and T introduced at time $t = 25$, and 65 h, respectively, each sustained for 10 h. A random noise is applied to the manipulated variables $C_{A,in}$. We assume both the disturbances in Q_{in} and T are not measured. As a result, the states x_1 are perturbed from their nominal values (especially so for the disturbance in T). The disturbance effect is passed through to the distillation column. However, y_2 ($x_{A,1}$) remains unaffected, because it is decoupled from the interconnection (from the reactor). Once the disturbance is removed, y_1 (C_A) reverts back to its nominal value (plus noise) almost immediately, as it is decoupled from the interconnection (from the distillation column). As the network system is locally internally stable, the other states remain stable and converge back to their nominal values once y_1 is at its nominal value.

Discussion

In this section, several important remarks about the results presented in the "Large-Scale Minimum Phase Analysis" and "Interconnection Decoupling" sections are discussed. In particular, we highlight the importance of minimum phase property to plantwide control, link the interconnection decoupling results to control structure selection, and compare the benefits and difficulties of distributed control against complete decentralized control.

Minimum phase property and plantwide control

The minimum phase property plays a very important role in control design. It essentially determines, to some extent, the difficulty in control design (especially for nonlinear case) and the limitations on the achievable control performance. The latter can be understood from the fundamentals of internal model control,[46] where it is widely known that the minimum phase property determines the stability of the inverse dynamics of a process[47,48] and thereby the possibility of a perfect control. The minimum phase property is also crucial

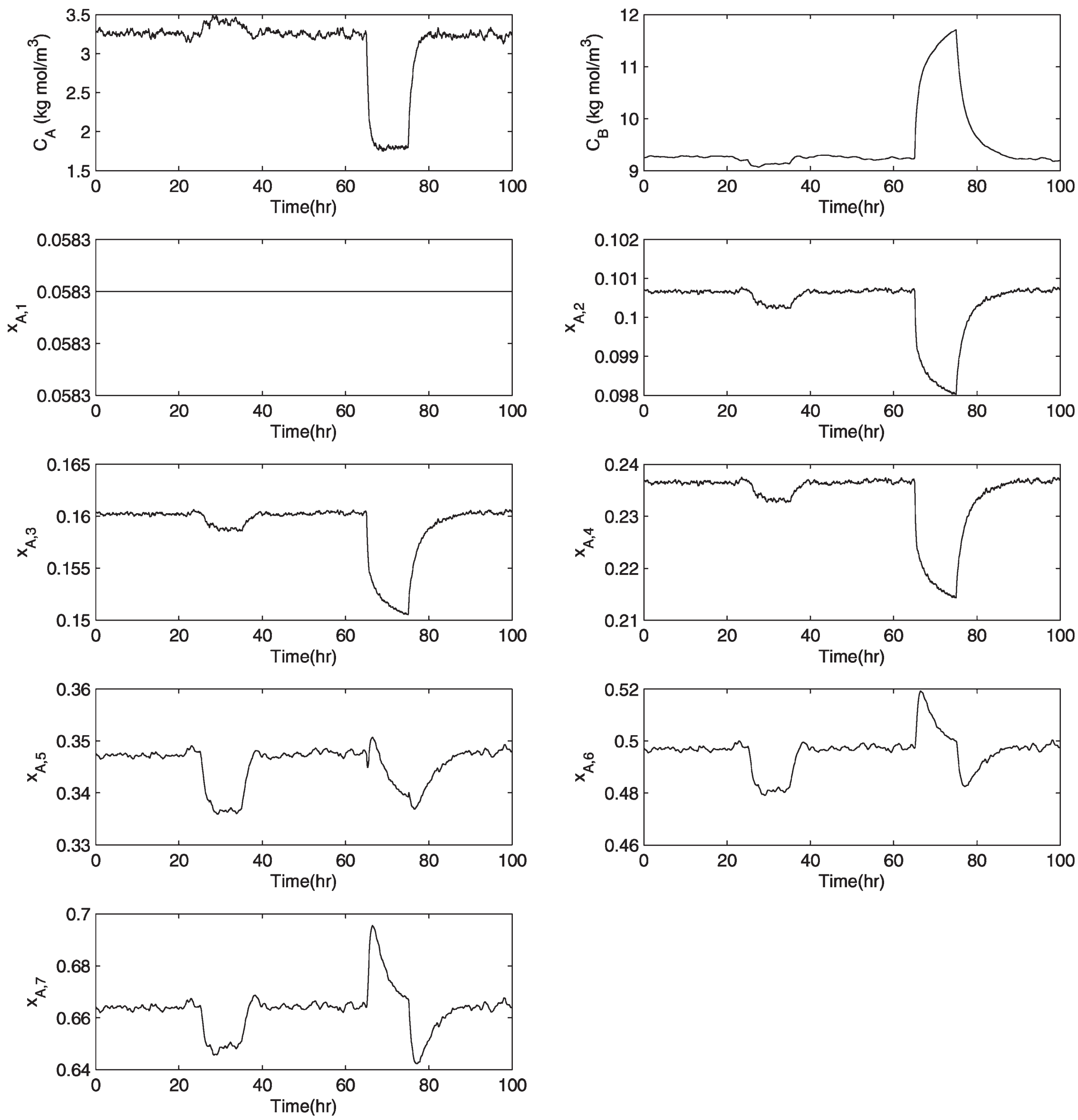

Figure 5. Dynamic response of the network system.

for constructive nonlinear control of underactuated systems based on geometric approaches[49] (e.g., via feedback linearization or recursive backstepping). Note that, according to conservation and thermodynamic laws, the state space of chemical process systems in general consists of multiple states[50] and in most plantwide applications, these systems are often underactuated (see, e.g., the case study by Downs and Vogel[51]). The usual practice is to use the degree of freedom available from the manipulated variables to control a subset of the state variables and to rely on the properties of the remaining zero dynamics to ensure the internal stability of the system. Note that while some other control techniques may have been successfully developed to control nonminimum phase systems (i.e., systems with unstable zero dynamics), it has been reported that instability may occur when these closed-loop systems are interconnected with other systems.

Notice that in our system description (6), the internal states of different subsystems are interconnected (through $\tilde{u}_i$). In the context of process control, this illustrates the scenario where the component mass and energy of different subsystems are linked (via $\tilde{u}_i$ and $\tilde{y}_i$)—which occurs in general (see, e.g., the case study by Downs and Vogel[51]) unless some component mass is filtered out to the environment or heat utilities are installed in between subsystems to regulate the energy flows. The main analysis results of this article derive a condition that determines the overall stability of these interacting internal dynamics when the output of each subsystem is successfully driven to the desired operating

Table 2. Distillation Operating Points

Stage	N	x_A
1 (reboiler)	2.50	0.06
2	0.25	0.10
3	0.25	0.16
4 (below feed)	0.25	0.24
5 (above feed)	0.25	0.35
6	0.25	0.50
7 (condenser)	2.50	0.66

point. In other words, the plantwide stability of zero dynamics (or plantwide minimum phase property) may be studied in this manner. Not much studies can be found on this topic in process control literature despite its importance. It is known that the interconnection of two stable systems may not be stable.[17] In the same manner, the interconnection of zero-output subsystems with stable zero dynamics may not be overall stable. Such analysis should be performed in the first step of control structure design activity (i.e., in the selection of controlled variables) alongside other optimization considerations.[2] Many previous developments on plantwide control design assume the above internal stability property and validate it via simulation results. Such simulation-based approaches lack rigorous conclusion and may not be scalable. In this article, a more systematic approach is proposed to derive such property through the use of dissipative systems theory, which is a generalization to the standard Lyapunov stability criteria.

Interconnection decoupling and control structure design

Interconnection decoupling is a condition where the controlled output of a system is isolated from the effects of interactions between subsystems. Therefore, the decoupling control law is descriptive of the interaction effects themselves. The amount of information required by the control law describes the scale of the interactions. For example, in Corollary 3, the interactions are local (each Σ_i is only dependent on the dynamics of local neighboring systems). The results of Theorem 4 shows that both the relative degree and the interconnection characteristic index can be used to determine the amount of distribution of information required to achieve interconnection decoupling. If each Σ_i satisfies $\rho_i < \nu_i$, it is shown that the output $y_i = z_{i,1}$ for each Σ_i is globally stabilizable to the equilibrium value (i.e., $y_i = 0$) under decentralized control action. In the case where $\rho_i \geq \nu_i$, almost interconnection decoupling may still be achieved under decentralized control strategy given that certain geometric restrictions are satisfied (see the article by Marino et al.[30]). This condition ensures that the $\mathcal{H}_\infty$ norm from $\tilde{u}_i$ to y_i is bounded, that is, $\|y_i\| \leq \eta_i \|\tilde{u}_i\|$ with $\eta_i > 0$. It is important to note, however, that such condition may not be trivially used for output stabilization of large-scale systems. To see this, consider the case where almost interconnection decoupling is possible for each Σ_i. Then, $\|y_i\| \leq \eta_i \|\tilde{u}_i\|$ with $\eta_i > 0$ for all $1 \leq i \leq N$. It follows that $y_i \to 0$ is only ensured if $\tilde{u}_i \to 0$. Recall that $\tilde{u}_i = \tilde{u}_i(z_1, \ldots, z_N, \xi_1, \ldots, \xi_N)$ with $\tilde{u}_i(0, \ldots, 0, 0, \ldots, 0) = 0$. Therefore, $\tilde{u}_i$ may not vanish before y_i (and ξ_i) of each Σ_i vanishes. Upon certain additional assumptions on $\tilde{u}_i$, however, the convergence $y_i \to 0$ can still be ensured. In many decentralized nonlinear control literature (see, e.g., the work by Han and Chen,[21] Guo et al.,[22] and Xie and Xie[23]), these limitations translates into the conditions on the bounds of $\tilde{u}_i$ (and the interconnection vector fields w_i). For example, Han and Chen[21] only allowed networks with strong interconnections (those which satisfy certain matching conditions). The results by Xie and Xie[23] require each subsystem to be minimum phase and the interactions between the zero dynamics have to vanish when the output $y_i = 0$ (note that in this case, the large-scale minimum phase property is trivially satisfied). We shall briefly remark, however, that regardless of the geometric properties of the system, the solution to output stabilizing problems may be found via algebraic (or other) methods. In general, however, such solution is not constructive and difficult to find. To summarize the above observations, we make the following statements.

The large-scale system (1) is stabilizable using decentralized control laws if the interconnections of the zero dynamics are stable and each Σ_i *has the property* $\rho_i < \nu_i$. *If* $\rho_i \geq \nu_i$ *for some* i, $1 \leq i \leq N$, *the large-scale system is stabilizable under decentralized control laws if the interconnections of the zero dynamics are stable and certain structural conditions about the subsystems and their interconnections are satisfied.*

For the purpose of this paper, we exclude the discussion on the many, specific geometric conditions that are required for constructive nonlinear decentralized control of different classes of large-scale systems. Readers are referred to the relevant references (e.g., the work by Han and Chen,[21] Guo et al.,[22] and Xie and Xie[23]) for more detailed developments. By allowing a distributed control structure, some of the above geometric limitations may be relaxed. By distributed control, we mean that the controller for each Σ_i has the access to the states in Σ_i and also in some other Σ_j, $1 \leq j \leq N$. One may interpret the distributed control structure as the transition from completely decentralized control structure to centralized control structure, discussed next.

At this point, it is worthwhile to mention that the concept of relative degree can also be used to study the achievable quality of control design due to the natural structure of a nonlinear system. Daoutidis and Kravaris[27] showed that the relative degree provides a measure of sluggishness of response and "physical" closeness (as defined by Seborg et al.[52] and Daoutidis and Kravaris[27]). Furthermore, it can be used to assess the coupling between manipulated and controlled variables. These facts can (and should) be used together with the results of this paper to find a suitable pair of manipulated/controlled variables (such that we have, e.g., minimal relative degrees). In this paper, we go beyond the concept of relative degree and suggest that the interconnection characteristic indices need to be considered as well to maintain the simplicity of control design, in view of large-scale plantwide control.

Decentralized and distributed control structures

From a geometric point of view, the benefits of decentralized control structure lie in its robustness to interconnection uncertainties—these usually resulted from unmodeled interaction dynamics and uncertain transport delays. The results by Han and Chen,[21] Guo et al.,[22] and Xie and Xie[23] show how robust nonlinear decentralized control law can be constructed only with the knowledge of the (nonlinear) norm upper-bound of interconnections. The main weakness of geometric nonlinear decentralized control structure, in the context of process control applications, is perhaps its lack of generality. In most of the problem formulations, the subsystems are required to possess certain geometric properties that are not necessarily satisfied by many process control systems. In many developments (e.g., by Guo et al.[22]), it requires each subsystem to have similar geometric properties (and even dimensionality). This can be limiting in plantwide process control where the subsystems in general satisfy different sets of geometric conditions (compare, e.g., a reactor and a distillation column). Some of these issues are of a less concern in distributed control structure, for example, with the help of hierarchical design rules. The robustness issue of distributed control structure is, however, still an open problem. In the above development (the proof to Theorem 4), this may be accomplished by recursive domination approach.[15]

Concluding Remarks

This article highlights the importance of interaction effects of internal dynamics in the context of plantwide nonlinear control systems. Conditions for large-scale minimum phase property are formulated using dissipation inequalities. These conditions reveal that the overall stability of large-scale systems (in the sense of Lyapunov) can be adversely affected by the interactions between the zero dynamics of their subsystems. An interconnection decoupling approach is proposed to design distributed feedback controllers that eliminate the effect of interactions on the controlled variables and guarantee their stabilization to the desired equilibrium values in the presence of interconnecting inputs. It was found that the relative degree and the interconnection characteristic index of the subsystems are crucial to the design of interconnection decoupling controllers. In particular, if the relative degree is smaller than the interconnection index, decentralized control laws are sufficient for interconnection decoupling. In the case when the relative degree is larger than the interconnection characteristic index, additional information from neighboring subsystems may be required to achieve interconnection decoupling. The analytic results presented in this article are general, however, for a large class of chemical systems, they can be numerically demonstrated for polynomial control systems through the use of sums of squares decomposition and linear programming techniques or linearization methods.

Our future work includes the development of analytic approaches to make the design and analysis more general and robust, for example, using dissipation inequalities. Recent developments in adaptive control and distributed model predictive control may also be integrated into the solutions.[15,53] Extensions of this work would consider the problem of plantwide control design using output feedback laws within the framework proposed here.

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